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DARK MATTER CAUSTICS AND THE ENHANCEMENT OF SELF-ANNIHILATION FLUX

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ABSTRACT

Cold dark matter haloes are populated by caustics, which are yet to be resolved in N-body simulations or observed in the Universe. Secondary infall model provides a paradigm for the study of caustics in *typical* haloes assuming that they have had no major mergers and have grown only by smooth accretion. This is a particular characteristic of the smallest dark matter haloes of about 10^{-5} $M_{\odot}$, which although *atypical* contain no substructures and could have survived until now with no major mergers. Thus using this model as the first guidline, we evaluate the neutralino self-annihilation flux for these haloes. Our results show that caustics could leave a distinct sawteeth signature on the differential and cumulative fluxes coming from the outer regions of these haloes. The total annihilation signal from the regions away from the centre can be boosted by about forty percents.

Subject headings: dark matter haloes, caustics, dark matter detection

1. INTRODUCTION

Evidence from the rotation curves of galaxies, gravitational lensing, microwave background radiation, peculiar velocity fields, and many other observations indicate that the visible mass, in the form of stars and hot gas, is only a small fraction of the total content of the Universe. The nature of the missing mass, the *dark matter*, remains unknown but is widely presumed to be Weakly Interacting Massive Particles (WIMPs), such as the lightest supersymmetric particles, which are yet to be detected in particle accelerators (Jungman, Kamionkowski, & Griest 1996 ; Bertone, Hooper & Silk 2004).

Accelerator searches are complemented by the vast experimental efforts to detect these particles in our galaxy and in nearby galaxies which are believed to be embedded in dark matter haloes (Ostriker & Peebles 1973). Such complementary techniques presently involve direct detection in low background laboratory detectors (Goodman & Witten 1985) and indirect detection through observation of energetic neutrinos, gamma rays and other products of self-annihilation of dark matter particles (Silk & Srednicki 1984) .

The event rate for self-annihilation depends quadratically on the local dark matter density, which falls off with distance from the center of the halo. The averaged halo density profile obtained in various numerical simulations diverges at the centre but is otherwise smooth and is often fit with a(n asymptotically) double power-law (Navarro, Frenk & White 1996, Moore et al 1998). However, a consensus on the precise values of the power exponents, the size of the central core and the resolution of fine high-density structures are yet to be achieved. The fine structures, the *caustics*, are inevitable outcomes of the evolution of a collisionless self-gravitating system described by the Jeans-Vlassov-Poisson equation (for a one-dimensional numerical result see Alard & Colombi 2005). Formally, in three dimensions, the most common caustics are surfaces of zero thicknesses over which the density diverges. [4] However, a maximum cut-off to their density is set by the finite non-negligible velocity dispersion of dark matter particles. Their density however remains very high and hence they can be significant for dark matter search experiments (Sikivie & Ipser 1992, Sikivie et al 1997, Natarajan 2007). The effect of velocity dispersion in the smearing of the caustics is expected to dominate over other effects such as particle discreteness which would also smooth the caustics but to a far lesser degree. Mergers of haloes can also smear out the caustics substantially and due to this fact we restrict our study to haloes that have grown by slow and smooth accretion. Nevertheless, caustics are robust, in that while they may break up into micro-caustics, they remain in the fine-scale halo substructure and thereby contribute to the general clumpiness boost of any annihilation signal.

Analytic studies of the formation of haloes and caustics have been carried out mainly under various simplifying assumptions, such as spherical symmetry, self-similarity, and cold and smooth accretion (Gott 1975, Gunn 1977, Fillmore & Goldreich 1984, Bertschinger 1985). In an Einstein-de Sitter Universe a spherical overdensity expands and then *turns around* to collapse. After collapse and at late times, the fluid motion becomes self-similar: its form remains unchanged when its length is rescaled in terms of the radius, $r_{\rm ta}$, of the shell that is currently at turn-around and is falling onto the

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[4] The general theory of singularities (Arnol'd, Shandarin& Zel'dovich 1982) also predicts singularities on lines and at points. Despite the greater concentration of mass in these singularities they probably play a less important role in the total annihilation rate because they contain a considerably smaller amount of mass. However, this has not been studied in detail.

galaxy for the first time. Physically, self-similarity arises because gravity is scale-free and because mass shells outside the initial overdensity are also bound and turn around at successively later times. Self-similar solutions give power-law density profiles on the scale of the halo which provides an explanation for the flattening of the rotation curves of galaxies. However, on smaller scales the density profile contains many spikes (*i.e.* caustics) of infinite density. The position and time of formation of these caustics are among the many properties that have been established in the framework of the self-similar infall model (Fillmore & Goldreich 1984, Bertschinger 1985).

In reality, dark matter has a small velocity dispersion and haloes do suffer from major mergers and non-sphericity. However, until numerical simulations achieve sufficient resolution, the self-similar accretion model provides a useful guideline to haloes which have not undergone major mergers.

Here, we use the self-similar model of halo formation and a further elaboration which includes the velocity dispersion of dark matter (Mohayaee & Shandarin 2006) as a first *guidline* to describe the evolution of the smallest haloes which have survived major merger and disruption until now and have grown only by slow accretion.

The application of self-similar model to such haloes can be viewed from two contradictory angles. One might assume that minihaloes are expected to be well-represented by this model, since they contain no substructures, have not undergone merger and grow very slowly only by smooth mass accretion. On the other hand, minihaloes are not typical haloes and selfsimilar accretion model is formulated to describe the evolution of a characteristic halo.

Keeping both of these issues in mind, we use self-similar model only as a first guidline for the evolution of minihaloes. A large number of them have been found in simulations (Diemand et al 2005). The simulations estimate the size of these haloes to be of about 0.01 pc (half mass radius) and their mass of $10^{-6}\mathrm{M}_\odot$ at $z = 26$. Due to resolution problems, these simulations are stopped at this redshift and typical evolution of galactic scale haloes is extended to minihaloes and the conclusion is drawn that about 10^{15} of these haloes could exist in the halo of MilkWay today. We assume that at least a fraction of these haloes have evolved by slow accretion model from $z = 26$ until now and use the selfsimilar model to evaluate their radius and mass at $z = 0$, which are respectively 1 pc and 10^{-5} $\mathrm{M}_\odot$. For these haloes and working self-consistently within our model including the contribution from the caustics, we demonstrate that in the outer regions of these haloes caustics can boost the annihilation signal by about 40%.

2. SECONDARY-INFALL MODEL WITH VELOCITY DISPERSION

The haloes considered in this work grow by smooth and slow accretion. A good example are the earth-mass haloes which were recently resolved (at z=26) in numerical simulations (Diemand et al 2005) and which although small are expected to have clean spherical caustics. We expect that at least a fraction of these haloes have survive disruption and major merger and grow by self-similar accretion model to a virial radius of about 1 pc and a mass of about 10^{-5} $\mathrm{M}_\odot$.

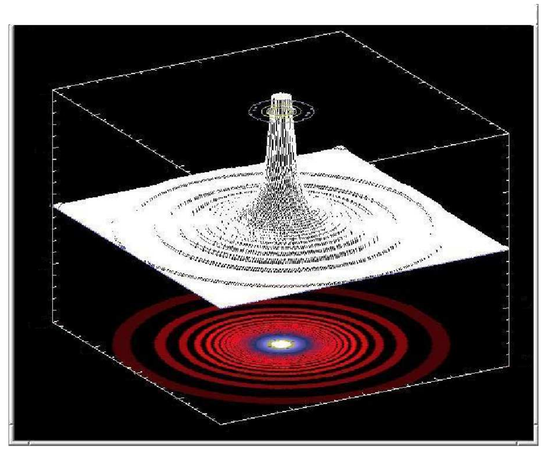

Fig. 1.— A surface-contour plot of the caustic density. In the self-similar model, caustics form concentric shells of increasing density and decreasing thicknesses and separations as we approach the center of the halo.

To comply with the requirement of slow accretion, we fix the value of the parameter ϵ in the initial density perturbation $\delta \sim M_i^{-\epsilon}$, where M_i is the initial mass, to unity. We emphasis that the self-similar model aims at describing the evolution of a typical halo. Typical haloes have mass variance ($\sigma(M)$ which varies as $M^{-(n+3)/6}$, which sets $\epsilon = (n+3)/6$, where n is the power spectrum index. A typical $\sigma(M)$ fluctuation grows as $t^{2/(3\epsilon)}$. Minihaloes correspond to the limit $n \to -3$ part of the spectrum. For this part of the spectrum, there are mass fluctuations of comparable amplitude on all scales and consequently adiabatic invariance does not apply for such fluctuations.

Hence, we use the self-similar model only as a first *guidline* for the growth of minihaloes. We assume that minihaloes of mass $10^{-6}M_\odot$ have grown by very small accretion from $z = 26$ to $z = 0$. Once again, slow accretion corresponds to the case of $\epsilon = 1$ in the work of Fillmore and Goldreich (1984), hence we shall adopt this value for ϵ.

The self-similar density profile is given by (Bertschinger 1985)

$$\frac{\rho}{\bar{\rho}} = \frac{\pi^2}{8\lambda^2}\sum_j (-1)^j \exp\left(-\frac{2}{3}\xi_j\right)\left(\frac{d\lambda}{d\xi}\right)_j^{-1} \qquad (1)$$

where $\bar{\rho}$ is the critical density and

$$\lambda = \frac{r}{r_{ta}} \qquad (2)$$

is the dimensionless radius and r is the physical radius and $\xi_i = \ln(t/t_{\mathrm{ta}})$ is the dimensionless time given in terms of the turnaround time, t_{ta}, of the particle that is at the jth point where $\lambda = \lambda(\xi)$ (see Bertschinger 1985 for further explanation).

The density (1) is evaluated numerically and plotted in Fig. 2 after an appropriate cut-off of the caustics which shall be discussed now. In principle the

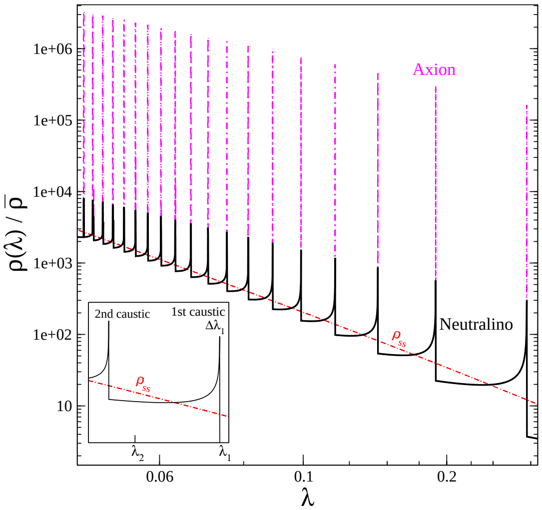

FIG. 2.— The plot is made by numerically solving (1) and cutting the caustics using (3). The density of halos (divided by the critical density $\bar{\rho}$) can be enhanced significantly at the caustics. This enhancement is far larger for axions (dotted violet spikes) than for neutralinos (continuous black line). The number of the streams increases to the center of the halo which explains the rapid growth of the smooth component of the density profile and the caustic contribution to the halo density diminishes. The dashed-dotted red line marked $\rho_{\rm ss}$ is the approximate self-similar density profile given by (6). The inset shows a magnified view of the second and third caustics for neutralinos.

density at the caustics diverges if the velocity dispersion of dark matter is zero. In the presence of a small velocity dispersion the maximum density and thickness of the caustic shells and their density profiles have been evaluated (Mohayaee & Shandarin 2006). The maximum density at the caustics and their profile are given by

$$\rho_{\rm caustic,k} = \begin{cases} \frac{G_k}{\sqrt{|\Delta\lambda_k|}}\bar{\rho} & \lambda_k - |\Delta\lambda_k| < \lambda < \lambda_k \\ \frac{G_k}{\sqrt{\lambda_k-\lambda}}\bar{\rho} & \lambda < \lambda_k - |\Delta\lambda_k| \end{cases} \tag{3}$$

where λ_k is the non-dimensional radius of the kth caustic counted inwards and

$$G_k = \frac{\pi^2}{4\sqrt{-2\lambda_k''}}\frac{e^{-2\xi_k/3}}{\lambda_k^2} \tag{4}$$

and the thickness of the caustic shell is given by

$$\Delta\lambda_k = \frac{(3\pi)^{2/3}e^{5\xi_k/9}\Lambda_k}{4}\,\frac{t\,\sigma(t)}{r_{\rm ta}}, \tag{5}$$

where t is the age of the Universe, σ is the present-day velocity dispersion of dark matter particles which is that at decoupling re-scaled with the expansion factor. The values of these parameters vary from one caustic to another (see Table 1 of Mohayaee & Shandarin 2006 for the first ten caustics). The profile (1) together with appropriate cut-off given by (3) is plotted in Fig. 2.

The peaked density profile given by (1) and shown in Fig. 2 has to be evaluated numerically. However, as is evident from Fig. 2 a "self-similar" profile[5] is reached which we fit with

$$\rho_{\sf ss} = \frac{2.8\lambda^{-9/4}}{(1+\lambda^{3/4})^2}\bar{\rho}\,, \tag{6}$$

as shown in Fig. 2 by the dashed-dotted red line, marked $\rho_{\sf ss}$. The turnaround radius, r_{ta} can now be evaluated by considering that at the virial radius the density is about 200 times the background density, and is given by $r_{\rm ta} \sim 4r_{\rm vir}$, which corresponds to the density profile given by (6). This approximate profile has been shown to be a good fit also to the mass profile (see Mohayaee & Shandarin 2006). In the next section we shall show that using this profile which ignores the caustics would yield an underestimated value for the flux.

Both the extrapolated numerical and the approximate density profiles shown in Fig. 2 formally diverge at the centre. However, due to finite dark matter velocity dispersion, haloes can develop central cores. Dark matter haloes are expected to have central cores due to the dark matter velocity dispersion, self-annihilations at the centre, angular momentum, tidal and various other effects. The core could be very small and the minimum scale associated with a generic dark matter merging history would conserve traces of the original cores in the initial substructure. These should be of order the free-streaming mass as for example computed in Bertschinger (2006).

In principle for small core sizes the total flux from the whole of the halo is dominated by the annihilation in the centre of the halo and the boost due to caustics is negligible. However, we shall show in the next section that the differential (similarly cumulative) flux would be distinctly marked by the caustics and shall have a sawteeth pattern and the contribution to the total flux from the outer region of these haloes by the caustics is significant and can yield a boost factor of about 40%.

3. THE FLUX DUE TO SELF-ANNIHILATION INCLUDING THE EFFECT OF CAUSTICS

Caustics if detected would be clear evidence of the existence of dark matter and could rule out alternative models of gravity. Two major methods for their detection are through gravitational lensing (see e.g. Gavazzi et al 2006) and the flux of dark matter annihilation product which is expected to be significantly enhanced by the caustics. Here we shall discuss the second method.

The flux of the self-annihilation product (*e.g.* γ-rays) is given by

$$\mathcal{F}lux \sim \int \rho^2(4\pi r^2)\,dr\,, \tag{7}$$

where the proportionality coefficient is a function of dark matter particle mass, interaction cross section and the number of photons produced per annihilation.

The differential and cumulative flux (*i.e.* the integrand in expression (7) and the integral evaluated from $r_{\rm ta}$ inwards) for neutralino ($\sigma = 0.03$ cm/s) and a minihalo of $r_{\rm ta} = 3.24$ pc (which corresponds to a virial radius of about 0.8 pc) is shown in Fig.

[5] This profile can also be well-fitted by a power-law and an exponential cut-off

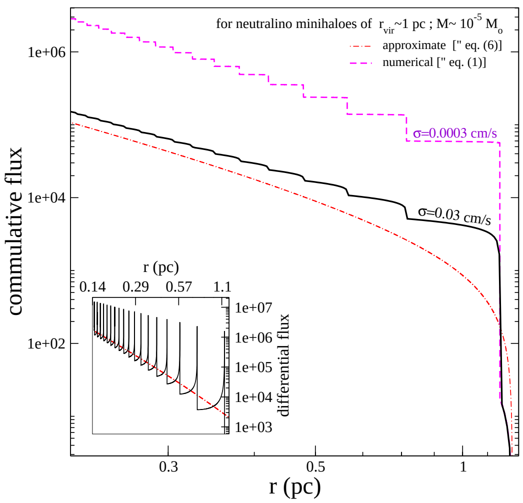

FIG. 3.— Cumulative flux is obtained by summing the flux inwards: *i.e.* from the first outer caustic towards the most inner (*i.e.* the integral (7) evaluated inwards). The flux is shown for two different values of the velocity dispersion. The red dashed line shows the cumulative flux obtained by using our approximate analytic expression for the density (6) which neglects the contribution from the caustics and can considerably underestimate the annihilation flux and ignore the distinct sawteeth characteristic of the caustics. The inset shows the differential flux (integrand of expression (7)) using the full density profile (1) as shown by the solid black spiky line and the approximate profile (6) as shown by the dashed-dotted red line. The sawteeth pattern is once again neglected in using the later profile.

3. The fluctuations, due to caustics, become less prominent as we go towards the centre. Decreasing the velocity dispersion would increase both the amplitude of the peaks in the density profile and the fluctuations in the flux, as shown in Figs. 2 and 3.

Using our numerical solution to (1) and approximation (6), we can now determine the flux from the neutralino minihaloes (Diemand et al 2005) and its enhancement due to the first twenty caustics. Clearly the total flux from the whole halo is dominated by the emission from the centre, where the density of the caustics reaches the background density (see also . However in the outer regions where the first twenty caustics dominate, as shown in Fig. 2 the ratio of the flux using the self-similar density profile given by (6) and the complete density profile (1) gives a boost factor of about

$$Boost = 1.4 \tag{8}$$

Thus, not only we expect a distinct signature on the cumulative and (similarly differential) flux due to caustics as highlighted schematically in Fig. 1 and shown numerically in Fig. 3, we also expect that the total flux from the outer halo region including the first twenty caustics to be boosted by about %40. Quantitative works on the gamma-ray flux is not carried out here, as it requires more realistic model than the self-similar model which can at best explain the growth of a typical halo. Minihaloes are atypical in the sense that they evolve in isolation, accreting almost no mass.

In conclusion, we have modeled dark matter haloes by an extended version of secondary infall model to include non-vanishing velocity dispersion. We have shown that the differential and cumulative fluxes would have distinct sawteeth pattern due to caustics. We have demonstrated that caustics can boost the total annihilation flux by about 40% percents in the outer regions of smallest haloes of about 10^{-5} $M_\odot$. As for the prospect of detecting caustics, the nearest minihaloes could be detectable in gamma rays by proper motions observed with GLAST (Koushiappas 2006), and should display a caustic-like substructure. One would expect to find a series of caustics, detectable as arclets. The predicted spacings could be used as a template to dig more deeply into the noisy background.

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